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A Model for the Pioneer Anomaly

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In a previous work we showed that massive test particles exhibit a non-geodesic acceleration in a modified theory of gravity obtained by a non-commutative deformation of General Relativity (so-called Matrix Gravity). We propose that this non-geodesic acceleration might be the origin of the anomalous acceleration experienced by the Pioneer 10 and Pioneer 11 spacecrafts.

1 Introduction

The Pioneer anomaly has been studied by many authors (see [1, 2, 11, 12, 8, 13] and the references in these papers) and it has a pretty strong experimental status [9]. It exhibits itself in an anomalous acceleration of the Pioneer 10 and 11 spacecrafts in the range of distances between 20AU and 50AU ($\sim 10^{14}$ cm) from the Sun. The acceleration is directed toward the Sun and has a magnitude of [1, 2]

$$A'_{\text{anom}} \approx (8.74 \pm 1.33) \times 10^{-8} \text{cm/s}^2. \quad (1.1)$$

In the last years there have been many attempts to explain the Pioneer anomaly by modifying General Relativity (see, for example, [11] and the references therein). However, there is also some evidence [13] that it could not be explained within standard General Relativity since it exhibits a *non-geodesic motion*. That is, it cannot be explained by just perturbing the Schwarzschild metric of the Solar system. It seems, from the analysis of the trajectories, that the spacecrafts do not move along the geodesics of any metric. Another puzzling fact is that there is no measurable anomaly in the motion of the planets themselves, which violates the equivalence principle. In other words, the heavy objects like the planets, with masses greater than $\sim 10^{27}$ g, do not feel any anomaly while the smaller objects, like the Pioneer spacecrafts, with masses of order $\sim 10^5$ g, do experience it.

There are also some interesting numerical coincidences regarding the Pioneer anomaly (noticed in [10] as well). Recall that the cosmological distance, which can be defined either by the Hubble constant H or by the cosmological constant Λ , is of order

$$r_0 \sim \frac{c}{H} \sim \frac{1}{\sqrt{\Lambda}} \sim 10^{28} \text{cm} \quad (1.2)$$

and the Compton wavelength of the proton is of order

$$r_1 \sim \frac{\hbar}{m_p c} \sim 10^{-13} \text{cm}. \quad (1.3)$$

Now, we easily see, first of all, that there is the following numerical relation

$$\left(\frac{r_1}{r_{\text{anom}}} \right) \sim \left(\frac{r_{\text{anom}}}{r_0} \right)^2, \quad (1.4)$$

where $r_{\text{anom}} \sim 10^{14}$ cm is the distance at which the anomaly is observed. This means that

$$r_{\text{anom}} \sim \left(\frac{\hbar}{m_p c \Lambda} \right)^{1/3} \sim \left(\frac{\hbar c}{m_p H^2} \right)^{1/3}. \quad (1.5)$$

Secondly, the characteristic distance determined by the value of the anomalous acceleration, $A_{\text{anom}} \sim 10^{-8} \text{cm/sec}^2$, is of the same order as the cosmological distance

$$r_2 \sim \frac{c^2}{A_{\text{anom}}} \sim 10^{28} \text{cm}, \quad (1.6)$$

which simply means that

$$A_{\text{anom}} \sim Hc \sim c^2 \sqrt{\Lambda}. \quad (1.7)$$

It is very intriguing to speculate that the *Pioneer effect is the result of some kind of interplay between the microscopic and cosmological effects at the macroscopic scales.*

In this paper we apply the investigation of motion of test particles in an extended theory of gravity, called Matrix Gravity, initiated in [6] to study the anomalous acceleration of Pioneer 10 and Pioneer 11 spacecrafts. Matrix Gravity was proposed in a series of recent papers [3, 4, 5]. This is a modification of the standard General Relativity in which the metric tensor $g^{\mu\nu}$ is replaced by a Hermitian $N \times N$ matrix-valued symmetric two-tensor $a^{\mu\nu} = g^{\mu\nu} \mathbb{I} + h^{\mu\nu}$, where \mathbb{I} is the identity matrix, $h^{\mu\nu}$ is a matrix-valued traceless symmetric tensor, i.e. $\text{tr } h^{\mu\nu} = 0$. In this theory the usual interpretation of gravity as Riemannian geometry is no longer appropriate. Instead, Matrix Gravity leads, quite naturally, to a generalized geometry, that we call Matrix Geometry, which is equivalent to a collection of Finsler geometries. Instead of a usual Riemannian geodesic flow, we get a system of Finsler flows, and, moreover, the mass of a test particle is replaced by a collection of mass parameters. In the commutative limit, only the total mass is observed. For more details and discussions see [4, 5, 6].

The dynamics of the tensor field $a^{\mu\nu}$ is described by a non-commutative Einstein-Hilbert action, which can be constructed either by an extension of all standard concepts of differential geometry to the non-commutative setting [3, 4] or from the spectral invariant of a partial differential operator of non-Laplace type [5].

The main goal of the present paper is to apply our previous study [6] of the motion of test particles (in a simple model of matrix gravity) to the Pioneer anomaly. We would like to stress that this study is just a first attempt to analyze the phenomenological effects of Matrix Gravity. We do not claim that this simple model definitely solves the mystery of the anomaly. Our aim is just to propose another candidate for its origin. Only future tests and more detailed models can describe the Pioneer anomaly in full capacity. This work does not represent the final answer, but just a first attempt of studying this phenomenon within the framework of Matrix Gravity.

2 Anomalous Acceleration in Matrix Gravity

In Matrix Gravity a massive particle is described not by a single mass parameter m but rather by N different mass parameters m_i , so that $m = \sum_{i=1}^N m_i$. In the commutative limit we only observe the total mass m . The interesting question of the physical origin of the parameters m_i requires further study. For this reason, we do not assume that the m_i are positive. Following [6] we consider two different cases. In the first case, that we call the *nonuniform model*, we assume that mass parameters are different, and in the second case, that we call the *uniform model*, we discuss what happens if they are equal to each other, that is, $m_i = m/N$.

The equations of motion of a test particle are derived and studied in [6]. They have the form

$$\frac{d^2 x^\mu}{dt^2} + \gamma^\mu_{\alpha\beta}(x, \dot{x}) \dot{x}^\alpha \dot{x}^\beta = 0, \quad (2.1)$$

where $\gamma^\mu_{\alpha\beta}(x, \dot{x})$ are generalized Christoffel coefficients that are homogeneous functions of \dot{x} of order zero, in other words, they depend on the direction of \dot{x} , but not on its magnitude. These functions depend in a complicated way on the matrix-valued metric $a^{\mu\nu}$, on the velocity, \dot{x}^μ , and, in general, on the ratios $\mu_i = m_i/m$.

In the perturbation theory, when one writes $a^{\mu\nu} = g^{\mu\nu} \mathbb{I} + h^{\mu\nu}$, the generalized Christoffel coefficients are

$$\gamma^\mu_{\alpha\beta}(x, \dot{x}) = \Gamma^\mu_{\alpha\beta}(x) + \theta^\mu_{\alpha\beta}(x, \dot{x}), \quad (2.2)$$

where $\Gamma^\mu_{\alpha\beta}$ are usual Christoffel coefficients of the metric of the metric $g_{\mu\nu}$ and $\theta^\mu_{\alpha\beta}(x, \dot{x})$ is some tensor of first order in the perturbation. Now, the matrix-valued metric $a^{\mu\nu} = g^{\mu\nu} \mathbb{I} + h^{\mu\nu}$ satisfies the non-commutative Einstein equations derived in [4, 5, 7]. As a result of non-commutative corrections even the equations for the metric $g^{\mu\nu}$ are modified. This means that both the metric $g^{\mu\nu}$ and the Christoffel symbols $\Gamma^\mu_{\alpha\beta}$ are modified. More precisely, we let

$$g^{\mu\nu} = g_0^{\mu\nu} + \beta^{\mu\nu}, \quad (2.3)$$

where $g_0^{\mu\nu}$ is the non-perturbed Riemannian metric given by the solution of the standard Einstein equations without any non-commutative corrections and $\beta^{\mu\nu}$ is the non-commutative correction. Then

$$\Gamma^\mu_{\alpha\beta} = \Gamma_0^\mu_{\alpha\beta} + \alpha^\mu_{\alpha\beta}, \quad (2.4)$$

where $\Gamma_0^\mu_{\alpha\beta}$ is the Christoffel symbols of the metric $g_0^{\mu\nu}$ and $\alpha^\mu_{\alpha\beta}$ is the perturbation.

Thus, the equations of motion take the form

$$\frac{d^2 x^\mu}{dt^2} + \Gamma_{0\alpha\beta}^\mu(x) \dot{x}^\alpha \dot{x}^\beta = A_{\text{anom}}^\mu(x, \dot{x}), \quad (2.5)$$

where

$$A_{\text{anom}}^\mu(x, \dot{x}) = A_{\text{geod}}^\mu(x, \dot{x}) + A_{\text{non-geod}}^\mu(x, \dot{x}) \quad (2.6)$$

is the anomalous acceleration and and

$$A_{\text{geod}}^\mu(x, \dot{x}) = -a_{\alpha\beta}^\mu(x) \dot{x}^\alpha \dot{x}^\beta, \quad (2.7)$$

$$A_{\text{non-geod}}^\mu(x, \dot{x}) = -\theta_{\alpha\beta}^\mu(x, \dot{x}) \dot{x}^\alpha \dot{x}^\beta. \quad (2.8)$$

are the geodesic and non-geodesic parts of the anomalous acceleration. It is this anomalous acceleration that we are going to study in this paper. We suggest that this might explain the anomalous behavior of Pioneer 10 and 11 spacecrafts.

The anomalous geodesic acceleration can be easily computed by expanding the Christoffel coefficients in the perturbation. In the first order in β we obtain

$$A_{\text{geod}}^\mu = \frac{1}{2} (2\nabla_\alpha \beta_{\beta}^\mu - \nabla^\mu \beta_{\alpha\beta}) \dot{x}^\alpha \dot{x}^\beta, \quad (2.9)$$

where the covariant derivatives and all tensor operations are performed with the non-perturbed metric $g_0^{\mu\nu}$.

The anomalous non-geodesic acceleration was derived within perturbation theory in the deformation parameter in [6]. We study the two cases mentioned above.

Nonuniform Model. First, we study the generic case when the parameters μ_i are different. We define a function

$$P(x, \xi) = \sum_{i=1}^N \mu_i \lambda_i(x, \xi), \quad (2.10)$$

where ξ_μ is a covector and $\lambda_i(x, \xi)$ are the eigenvalues of the matrix $h^{\mu\nu}(x) \xi_\mu \xi_\nu$. Note that since $\text{tr } h^{\mu\nu} = 0$ the matrix $h^{\mu\nu} \xi_\mu \xi_\nu$ is traceless, which implies that the sum of its eigenvalues is equal to zero. Thus, in the uniform case, when all mass parameters μ_i are the same, the function $P(x, \xi)$ vanishes. In this case the effects of non-commutativity are of the second order.

The non-geodesic acceleration was computed in [6] and has the form

$$A_{\text{non-geod}}^\mu = \frac{1}{2} g^{\mu\nu} (2\nabla_\alpha q_{\beta\nu}(x, \dot{x}) - \nabla_\nu q_{\alpha\beta}(x, \dot{x})) \dot{x}^\alpha \dot{x}^\beta, \quad (2.11)$$

where

$$q^{\mu\nu}(x, \xi) = \frac{1}{2} \frac{\partial^2}{\partial \xi_\mu \partial \xi_\nu} P(x, \xi) \quad (2.12)$$

and the covariant derivatives are defined with the Riemannian metric. Thus, the total anomalous acceleration is

$$A^\mu_{\text{anom}} = \frac{1}{2} \left[2\nabla_\alpha (\beta^\mu_\beta + q^\mu_\beta) - \nabla^\mu (\beta_{\alpha\beta} + q_{\alpha\beta}) \right] \dot{x}^\alpha \dot{x}^\beta. \quad (2.13)$$

Uniform Model. Now, we will simply assume that all mass parameters are equal, that is, $m_i = \frac{m}{N}$. The non-geodesic acceleration, computed in [6], has the form

$$A^\mu_{\text{non-geod}} = -\frac{1}{8} g^{\mu\nu} \left(2\nabla_\alpha S_{\beta\nu\rho\sigma} - \nabla_\nu S_{\alpha\beta\rho\sigma} \right) \dot{x}^\rho \dot{x}^\sigma \dot{x}^\alpha \dot{x}^\beta, \quad (2.14)$$

where

$$S^{\mu\nu\alpha\beta} = \frac{1}{N} \text{tr} (h^{\mu\nu} h^{\alpha\beta}). \quad (2.15)$$

Thus, the total anomalous acceleration is

$$A^\mu_{\text{anom}} = \frac{1}{2} \left(2\nabla_\alpha \beta^\mu_\beta - \nabla^\mu \beta_{\alpha\beta} \right) \dot{x}^\alpha \dot{x}^\beta - \frac{1}{8} g^{\mu\nu} \left(2\nabla_\alpha S_{\beta\nu\rho\sigma} - \nabla_\nu S_{\alpha\beta\rho\sigma} \right) \dot{x}^\rho \dot{x}^\sigma \dot{x}^\alpha \dot{x}^\beta. \quad (2.16)$$

In the spherically symmetric background, in the non-relativistic limit, the radial anomalous acceleration is given by: in the uniform model,

$$A^r_{\text{anom}} = \frac{\partial}{\partial r} \left(-\frac{1}{2} \beta^{00}(r) + \frac{1}{8} S^{0000}(r) \right), \quad (2.17)$$

and, in the nonuniform model,

$$A^r_{\text{anom}} = \frac{\partial}{\partial r} \left(-\frac{1}{2} \beta^{00}(r) - \frac{1}{2} q^{00}(r) \right). \quad (2.18)$$

Of course, this can also be interpreted as a modification of Newton's Law [6].

Here, of course, the tensor components β^{00} , S^{0000} and q^{00} should be obtained as solution of the non-commutative Einstein field equations (in the perturbation theory). These equations are somewhat complicated. That is why, in this paper we consider a *toy model* just to get a glimpse into the phenomenon.

We consider a simple model of 2×2 real symmetric commutative matrices. The static spherically symmetric solution of the matrix Einstein equations for this

model was obtained in [6]. In the spherical coordinates $x^0 = t$, $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$ it has the form

$$a^{00} = A, \quad a^{11} = B, \quad (2.19)$$

$$a^{22} = \frac{1}{r^2} \mathbb{I}, \quad a^{33} = \frac{1}{r^2 \sin^2 \theta} \mathbb{I},$$

with

$$B(r) = \left(1 - \frac{1}{3}\Lambda r^2 - \frac{r_g - \theta L}{r}\right) \mathbb{I} + \frac{L}{r} \tau, \quad (2.20)$$

$$A(r) = \varphi(r) \mathbb{I} + \psi(r) \tau,$$

where $\mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\tau = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, Λ is the cosmological constant, $r_g = 2GM$, M is the mass of the Sun, θ and L are some integration parameters, (the parameter θ should not be confused with the angle variable θ above), and $\varphi(r)$ and $\psi(r)$ are some functions (the function $\varphi(r)$ should not be confused with the angle variable φ above). The functions $\varphi(r)$ and $\psi(r)$ can be parametrized by

$$\varphi(r) = -f_1(r) + 2\theta f_2(r), \quad \psi(r) = \theta f_1(r) + (1 + \theta^2) f_2(r). \quad (2.21)$$

By introducing the following function

$$u(r) = 1 - \frac{1}{3}\Lambda r^2 - \frac{r_g}{r}, \quad (2.22)$$

we obtain, for $f_1(r)$ and $f_2(r)$ in (2.21), the expressions

$$f_1(r) = \frac{u(r)}{\left[u(r) + (\theta + 1)\frac{L}{r}\right]\left[u(r) + (\theta - 1)\frac{L}{r}\right]}, \quad (2.23)$$

$$f_2(r) = \frac{\frac{L}{r}}{\left[u(r) + (\theta + 1)\frac{L}{r}\right]\left[u(r) + (\theta - 1)\frac{L}{r}\right]}. \quad (2.24)$$

Notice that when the parameters θ and L vanish the functions $\varphi(r)$ and $\psi(r)$ give nothing but the standard Schwarzschild solution with the cosmological constant [6]. We take the standard Schwarzschild solution (without the cosmological

constant) as the non-perturbed metric $g_0^{\mu\nu}$. The parameters θ , L and Λ will be considered as perturbation. This means that the relevant components have the form

$$\beta^{00} = \varphi(r) + \frac{1}{1 - \frac{r_g}{r}}, \quad (2.25)$$

$$h^{00} = \psi(r)\tau, \quad (2.26)$$

and, therefore,

$$S^{0000} = \psi^2(r). \quad (2.27)$$

In this simple 2×2 model the tensor $q^{\mu\nu}$ is given by $q^{\mu\nu} = \frac{\gamma}{2} \text{tr}(h^{\mu\nu}\tau)$, [6], where $\gamma = \mu_1 - \mu_2$, and therefore,

$$q^{00} = \gamma\psi(r). \quad (2.28)$$

As we already mentioned above, in the non-relativistic limit the only essential component of the anomalous acceleration is the radial one A_{anom}^r . By using the equations above we obtain: in the uniform model,

$$A_{\text{anom}}^r = \frac{1}{2} \frac{\partial}{\partial r} \left[-\frac{1}{1 - \frac{r_g}{r}} + f_1(r) - 2\theta f_2(r) + \left(\frac{\theta}{2} f_1(r) + (1 + \theta^2) f_2(r) \right)^2 \right], \quad (2.29)$$

and in the non-uniform model,

$$A_{\text{anom}}^r = -\frac{1}{2} \frac{\partial}{\partial r} \left[\frac{1}{1 - \frac{r_g}{r}} + (\gamma\theta - 1)f_1(r) + \gamma(1 + \theta^2)f_2(r) \right]. \quad (2.30)$$

We would like to emphasize at this point that the perturbation theory is only valid for small corrections. Obviously, when the corrections become large one needs to consider the exact equations of motion.

3 Pioneer Anomaly

We have two free parameters in our model, θ and L (and γ in the non-uniform model). We estimate these parameters to match the value of the observed anomalous acceleration of the Pioneer spacecrafts.

First of all, we recall the observed value of the cosmological constant $\Lambda \approx 2.5 \cdot 10^{-56} \text{cm}^{-2}$; therefore, $r_0 \approx |\Lambda|^{-1/2} = 6.3 \cdot 10^{27} \text{cm}$, and the gravitational radius of the Sun $r_g \approx 1.5 \cdot 10^5 \text{cm}$. The relevant scale of the Pioneer anomaly is $r_{\text{anom}} \sim 10^{14} \div 10^{15} \text{cm}$, therefore, we can restrict our analysis to the range

$r_g \ll r \ll r_0$. The values of the dimensionless parameters are $\frac{r_g}{r} \sim 10^{-8}$, $\frac{r}{r_0} \sim 10^{-15}$, and $\frac{r_g}{r_0} \sim 10^{-23}$. We also remind that the value of the anomalous acceleration is $A_{\text{anom}}^r \approx 8.7 \cdot 10^{-8} \text{cm/s}^2$. We should stress that our analysis only applies to the range of distances relevant for the study of the Pioneer anomaly. Therefore, strictly speaking, from a formal point of view, one cannot extrapolate our equations beyond this interval. Since the parameters $\frac{r_g}{r}$, $\frac{r}{r_0}$ and $\frac{r_g}{r_0}$ are negligibly small (compared to 1) they can be omitted.

By using the eqs. (2.29) and (2.30), and by defining $\rho = (1 + \theta^2)L - \theta r_g$, we obtain [6] (in the usual units, c being the speed of light) for $r_g \ll r \ll r_0$: in the uniform model,

$$A_{\text{anom}}^r = -\frac{c^2}{4} \left(\theta + \frac{\rho}{r} \right) \left(\frac{\rho + 2\theta r_g}{r^2} - \frac{2}{3} \theta \Lambda r \right), \quad (3.1)$$

and in the non-uniform model,

$$A_{\text{anom}}^r = \frac{c^2}{2} \gamma \left(\frac{\rho + 2\theta r_g}{r^2} - \frac{2}{3} \theta \Lambda r \right). \quad (3.2)$$

Uniform Model. First, we restrict to the case of vanishing cosmological constant. Then the function (3.1) takes the form

$$A_{\text{anom}}^r(r) = -\frac{c^2}{4} \left(\theta + \frac{\rho}{r} \right) \frac{\rho + 2\theta r_g}{r^2}. \quad (3.3)$$

It has an extremum if the signs of θ and ρ are different, which occurs at $r_* = -\frac{3}{2} \frac{\rho}{\theta}$ and is equal to

$$A_{\text{anom}}^r(r_*) = -\frac{c^2 \theta^3 (\rho + 2\theta r_g)}{27 \rho^2}. \quad (3.4)$$

Now, we assume that $r_* \sim r_{\text{anom}} \sim 10^{14} \text{cm}$ and $A_{\text{anom}}^r(r_*) \sim -10^{-8} \text{cm/sec}^2$ to estimate the parameters

$$\rho \sim 10^7 \text{cm}, \quad \theta \sim -10^{-7}. \quad (3.5)$$

If we leave the cosmological constant there is another range of parameters that should be investigated. Namely, when the term $\frac{2\theta}{3r_0^2}r$ becomes comparable with the term $\frac{\rho}{r^2}$. In this case the anomalous acceleration can be written, by dropping negligible terms, as

$$A_{\text{anom}}^r(r) = -\frac{c^2}{4r_0} \left(\frac{\theta \rho r_0}{r^2} + \frac{2\theta^2}{3r_0} r \right). \quad (3.6)$$

We note that the term $\frac{c^2}{4r_0}$ gives the right magnitude of the anomalous acceleration. If we assume that the two terms in the parentheses are comparable at the characteristic length r_{anom} and are of order 1, then we get an estimate

$$\rho \sim \frac{r_{\text{anom}}^3}{r_0^2} \theta \quad \text{and} \quad \theta \sim \left(\frac{r_0}{r_{\text{anom}}} \right)^{\frac{1}{2}}, \quad (3.7)$$

and, therefore,

$$\rho \sim 10^{-7} \text{cm} \quad \text{and} \quad \theta \sim 10^7. \quad (3.8)$$

Nonuniform Model. In the non-uniform model we have an additional parameter γ . The function has an extremum at

$$r_* = \left(\frac{3\rho r_0^2}{\theta} \right)^{1/3}. \quad (3.9)$$

Now, we assume that $r_* \sim r_{\text{anom}} \sim 10^{14} \text{cm}$; then

$$\frac{\rho}{\theta} = \frac{r_*^3}{3r_0^2} \sim 10^{-13} \text{cm}. \quad (3.10)$$

Further, by assuming $A^r_{\text{anom}}(r_*) \sim -10^{-8} \text{cm/sec}^2$ and using the eq. we estimate the parameter γ

$$\gamma \sim 10^{13}. \quad (3.11)$$

It is interesting to notice that, in this case, ρ/θ has the same order of magnitude of the Compton wavelength of the proton. Moreover, by using (3.10), we confirm the coincidence (1.5) mentioned in the introduction. This is very intriguing; it allows one to speculate that the anomalous acceleration could be a result of an interplay between the microscopic and macroscopic worlds, in other words, the Pioneer anomaly could be a quantum effect.

4 Conclusions

In this paper we applied the kinematics of test particles [6] in Matrix Gravity [4, 5] to the study of Pioneer anomaly. Matrix Gravity is interpreted in terms of Matrix Geometry, a generalized geometry which is equivalent to a collection of Finsler geometries, rather than Riemannian geometry. This new feature of our theory leads to an interesting and completely new phenomenon of splitting of Riemannian geodesics to a collection of Finsler geodesics. More precisely, instead of one

Riemannian metric we have different Finsler metrics and different mass parameters which describe the tendency to follow a particular trajectory determined by a particular Finsler metric. The interesting result is that test particles in our theory exhibit a non-geodesic motion which can be interpreted in terms of an anomalous acceleration. This new feature led us to apply these results for studying the anomalous acceleration of the Pioneer spacecrafts.

We considered two models: a uniform one, in which a particle is described by a single mass parameter, and a non-uniform one, in which a particle is described by multiple mass parameters. The choice of one model over the other should be dictated by physical reasons. The interesting question of whether the matter is described by only one mass parameter or more than one mass parameters requires further study. If the Pioneer anomaly is a new physical phenomenon we have to accept the fact that the equivalence principle does not hold. If this is the case, a model with different mass parameters (violating the equivalence principle) would be more appropriate to describe the motion of test particles in the Solar system.

The next step of our analysis of the phenomenological consequences of Matrix Gravity is to apply the kinematic model developed in [6] to the study of galactic rotations. It would be very interesting to understand if the flat rotation curves of galaxies can be explained without the concept of dark matter. We plan to investigate this question in a future work.

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